Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra, by Jiri Matousek

This book is not only an overview or a beginner's textbook (in fact, its structure is somewhat anti-textbook), but a calling invitation into the world of combinatorics. Not only in the world of combinatorics, also in that of linear algebra. And not even only in these two subjects: number theory, algorithms and codes, geometry will appear on the pages. The pages that carry mathematical gems, not just essays, turning the book into a small but rich enough placer. Everyone can discover it to ones personal admiration.

Each of the thirty-three miniatures is self-contained, although some of them refer to others, however different entries require different mathematical background from the reader. These miniatures are sometimes excerpts of known uses of linear algebra in other areas of mathematics, sometimes they are a little sketchy, so a certain level of mathematical culture is necessary in order to fully enjoy the reading. The book starts with simple but lively observation concerning the Fibonacci numbers and then soon involves way more complicated matters, not only classical results, but also contemporary ones. Every time the problem in question does not lie directly in the heart of linear algebra, but it has a solution emerging from there. Sometimes this solution is a well-known algorithm or a classical trick, showing just one of several possible approaches. Sometimes linear algebra comes in use unexpectedly and provides a really elegant and laconic solution, demonstrating its full power and beauty. In such a case, if the problem is a gemstone, then the methods of linear algebra are both a chisel and a scaif, giving the gem its cut and polishing it to perfection.

The book provides a deep insight on how far linear algebra protrudes in graph theory through the theory of graph spectra, by describing, e.g. the sparsest cut algorithm or the classification of Moore graphs. It also shows classical applications of linear algebra to extremal set theory, error-correcting codes, such as Hamming's code, and algorithms from graph theory, such as detecting sub-graphs of given type. Another big part is the description of several probabilistic algorithms, which are basically employed in verification of identities, although the applications shown are quite pictorial, such as efficient verification of matrix multiplication or testing the existence of a perfect matching in a graph. The book also shows the use of linear algebra in geometry, e.g. by considering equilateral sets or equiangular lines or tiling of a rectangle by squares.

Reading this book may be recommended to those having initial background in linear algebra, including undergraduates, and interested in combinatorics, geometry and associated computational algorithms. It will reward everyone seeking lively and interesting examples of the interplay between these parts of mathematics, both for learning or teaching needs, out of one's own curiosity or in order to illustrate one's future linear algebra course. The book will also be a perfect stimulating reading for those who want to broaden their general mathematical culture. The references provided in the preface and after each miniature give a good scope for further reading.

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